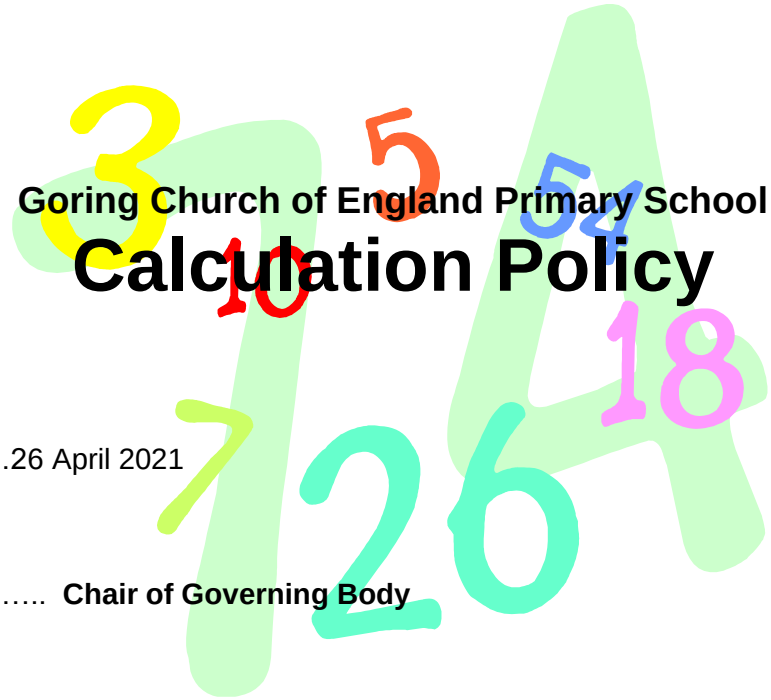
We are all part of God's vine and are rooted in His rich soil.
We are nurtured and supported so that we may grow and spread out into the world to love and to serve.

I am the vine; you are the branches.
If you remain in me and I in you, you will bear much fruit.
John 15:5



Goring Church of England Primary School

Calculation Policy

Policy Approved by the Governing Body on …26 April 2021

Signed …….. **Chair of Governing Body**

Review Date …April 2023

Introduction:
The aim of this policy is to ensure all members of our school community understand how calculations are taught and the progression through the stages of developing fluency with written methods. This will ensure calculations are taught, explained, and understood in a way that is systematic and consistent.

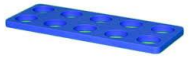
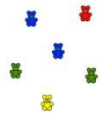
The National Curriculum (2013) for maths is based on 3 key aims:

- Pupils' developing fluency in the fundamentals of mathematics so that they develop conceptual understanding and the ability to recall and use maths rapidly and accurately
- Being able to use reasoning and generalisation to develop an argument or proof
- Developing pupils' ability to solve problems by applying maths to a range of increasingly complex problems.

At Goring Church of England Primary School, children are introduced to the processes of calculation through practical, oral and mental activities. As they begin to understand the underlying ideas, they develop ways of recording to support their thinking and calculation methods, so that they develop both **conceptual understanding** and **fluency** in the fundamentals of mathematics. Whilst interpreting signs and symbols involved with calculation, orally in the first instance, children use both manipulatives as well as pictorial representations (as part of a **Concrete-Pictorial-Abstract – CPA – approach**) to support their mental and written methods of calculation. As children's mental methods are strengthened and refined, they begin to work more efficiently, which will support them with using succinct written calculation strategies as they are developed. Both addition and subtraction, and multiplication and division are taught with an interconnectedness, as opposed to in isolation from each other.

From Early Years to Year 1:

There are fundamental concepts that it is important for children to develop an early understanding of as building blocks to future learning in maths, including that linked to calculation. A selection of the skills include:

- Ordinality – 'the ordering of numbers in relation to one another' – e.g. (1, 2, 3, 4, 5…)
- Cardinality – 'understanding the value of different numbers' – e.g. (7 =  17 =  +  14 = 
- Equality – 'seven is the same total as four add three' – e.g.  = 
- Subitising – 'instantly recognising the number of objects in a small group, without counting them' – e.g.  → five
- One-to-one correspondence – e.g. 
- Conservation of number – 'recognising that a value of objects are the same, even if they are laid out differently' – e.g.  
- Concept of zero  3 + 0 = 3
- Counting on and back from any number – e.g. 'five add three more totals eight'   'ten take away three totals seven'

The ability to calculate mentally forms the basis of all methods of calculation so they need to be explicitly taught and maintained. A good knowledge and 'feel' for numbers, is the product of structured practice through progression in relevant practical maths experiences alongside visual representations.

By the end of Year 6, children should be equipped with efficient mental and written calculation methods, which they use fluently. Decisions about when to progress should always be based on the security of pupils' understanding and their readiness to move ahead to the next stage. At whatever stage in their learning, and with whatever written method is being used, children's strategies must still be underpinned by a secure understanding and knowledge of number facts that can be recalled fluently with flexibility.

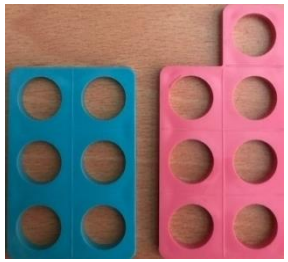
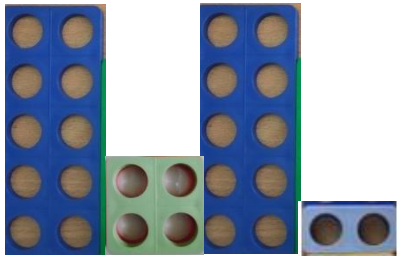
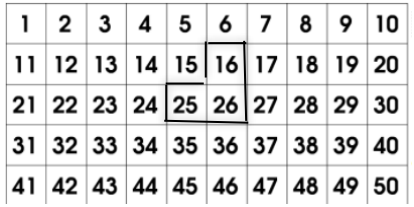
The overall aims are that when children leave primary school they:

- Are able to recall number facts with fluency, having developed conceptual understanding through being able to visualise key ideas – such as those related to place value, through experience with practical equipment and visual representations;
- Make use of diagrams (including the bar model) and jottings to help record / reason through stages of thinking when using mental methods that generate more information than can be kept in their heads;
- Have an efficient, reliable, written method of calculation for each number operation that they can apply with confidence when undertaking calculations that they cannot carry out mentally;
- Are able to make connections between all four number operations, understanding how they relate to one another, as well as how the rules and laws of arithmetic can be applied.

Mental calculation strategies for addition and subtraction:

All mental calculation strategies need to be taught explicitly using a Concrete – Pictorial – Abstract (CPA) approach in every year group, for example, using whole numbers in Key stage one and decimals in Key Stage 2. The following ideas can be adjusted so that they are accessible to all children.

The NCETM, 2015, states that, 'a pupil really understands a mathematical concept, idea or technique if he or she can represent it in a variety of ways.'

<table>
<tr>
<td>Doubles: 8 + 8 = 16
 
8 + 8 is connected to 8 X 2
  </td>
<td>Near doubles: 6 + 7 = 13
 
6 + 7 is commutative with 7 + 6
  </td>
<td>Number bonds: 7 + 3 = 10
 
  </td>
</tr>
<tr>
<td>Partitioning: 14 + 12 = 26
 
  </td>
<td>Bridging: 7 + 5 = 12
 
To begin: 7 + 3 = 10
Then: 10 + 2 = 12
 
  </td>
<td>Adjusting: 16 + 9 = 25
To begin: 16 + 10 = 26
 
Then: 26 – 1 = 25
 
  </td>
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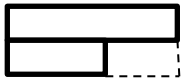
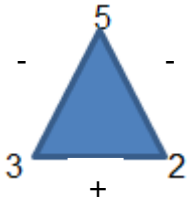
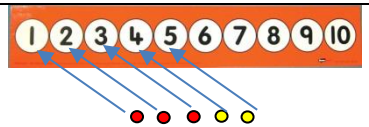
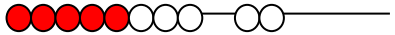
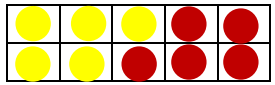
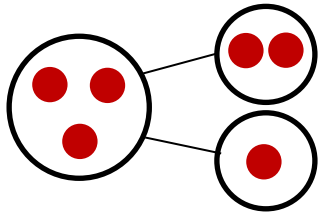
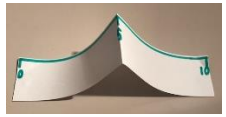
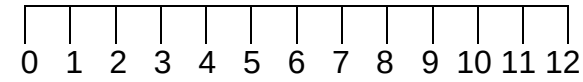
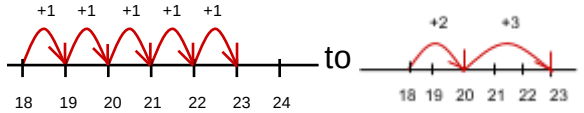
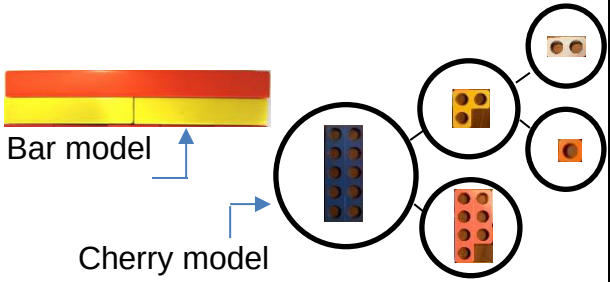
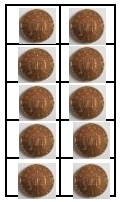
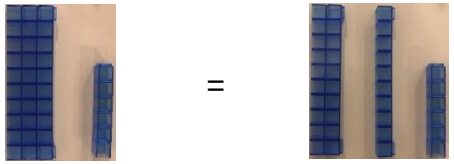
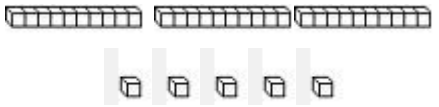
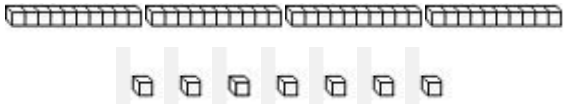
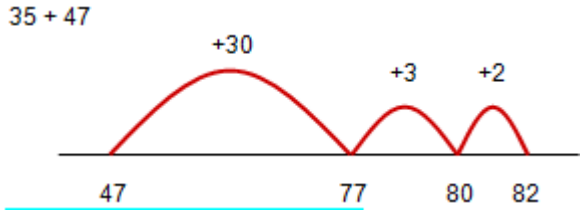
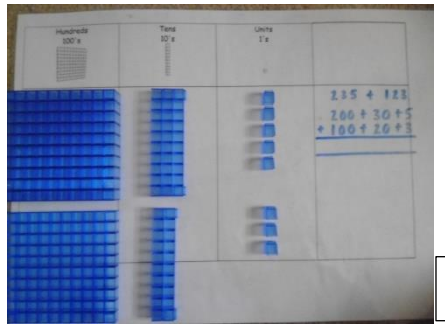
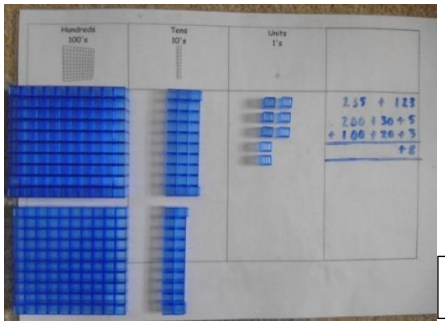
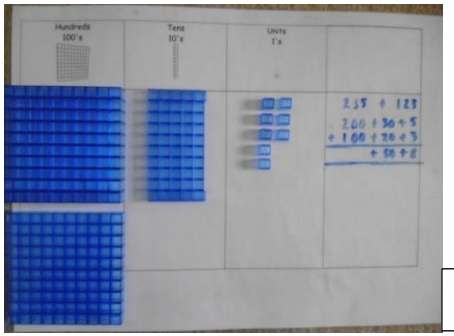
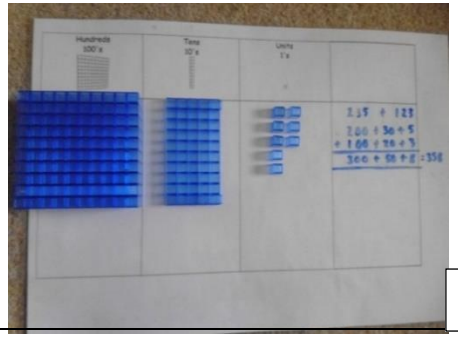
Finding the difference: 10 – 6 = 4   David has 10 sweets, whilst Chloe has six sweets. How many more does David have than Chloe? 	**Reordering:** 8 + 7 + 2 = 17 e.g. calculating numbers in a different order To begin: 8 + 2 = 10 Then: 10 + 7 = 17 	

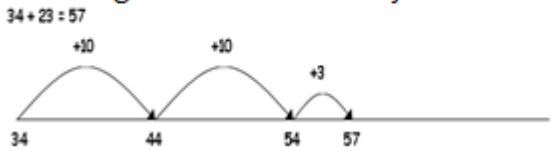
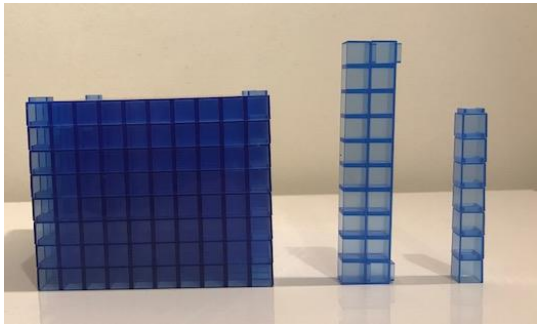
Addition:

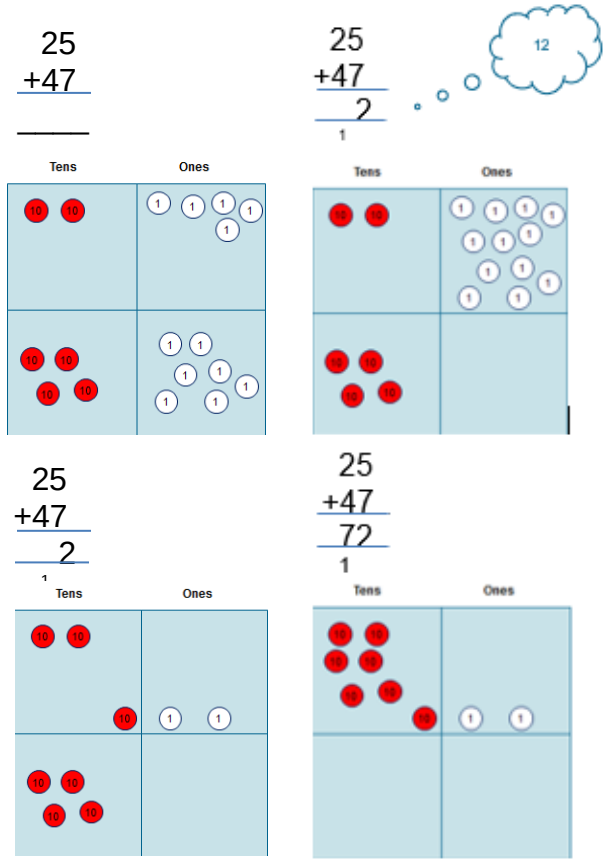
	Counting	Mental maths strategies & linked concepts	Rapid recall	Written calculation & appropriate models/images to support conceptual understanding	
Stage 1:	Count in ones to and across 100 forwards and backwards starting from 0, 1 and other numbers. Count in multiples of two, five and ten using a counting stick set up as a number track.	<u>Explicitly teach every mental maths strategy detailed above.</u> Pupils use apparatus to explore addition as the inverse of subtraction.  'Four add one is the same as five' 	Rapid recall of all pairs of numbers totalling numbers up to 20. Use structured apparatus – i.e. Numicon, tens frames, abaci, etc. 	**Combining two groups:** • Teachers model how to line up counters/objects on a number track before counting on. This is a precursor to use of a fully numbered number-line. • Children develop a mental picture of the number system for use with calculation. A range of key models and images support this, alongside practical equipment. **Whole / part-whole model:** • The concept of a whole / part-whole model is introduced.	 3 + 2 = 5 'Three plus two is the same as five'  'Eight add two more makes ten'  'Four add one more is the same as five'  Tens frame  Bar model  Cherry model

Stage 2:	Continue practising above skills. Count in steps of 2, 3 and 5 forwards and backwards to and from zero using a counting stick set up as a number line. Count in tens from any number – link to coins in a piggy bank as well as a number square.	<u>Explicitly teach every mental maths strategy detailed above.</u> Round numbers to the nearest 10, for example, by illustrating on a number line that is drawn on a folded strip of paper. 	Recall addition facts for all numbers to 20.	**Counting on from the largest number:** • Children begin to use number lines to support their own calculations, initially counting on from the largest number in ones before beginning to work more efficiently. **Reordering calculations to apply use of mental maths strategies:** • Children reorder 'strings' of numbers to apply their understanding of mental maths strategies, including doubles and number bonds, e.g. 6 + 7 + 4 reordered to 6 + 4 = 10 and then 10 + 7 = 17. Jottings are used to help keep track of thinking. **Whole / part-whole model:** • The concept of a whole / part-whole model is reinforced and extended.	Number line with all numbers labelled  18 + 5   Questions such as: 'How might I rearrange these to find the total?' are asked. 

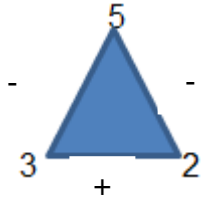
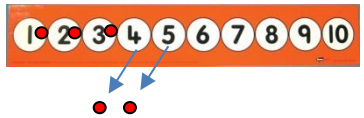
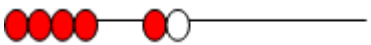
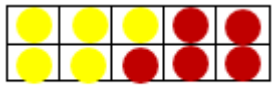
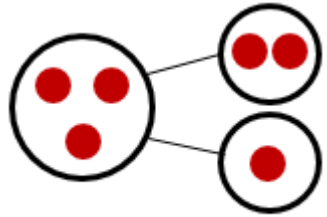
Stage 3:	Continue practising above skills. Count forward and backwards from 0 in multiples of 4, 8, 50 and 100. Count on 10 or 100 from any two-digit number. Count up and down in tenths. Link to a counting stick as before, whilst deriving number facts.	Reinforce partitioning and bridging through multiples of 10, plus adjusting when adding 11 or 9. Use structured apparatus to understand that subtraction undoes addition and link with inverse number operations.	Connect pairs totalling ten to pairs of multiples of 10 totalling 100.  Use 10ps in tens frame. Recall pairs of two-digit numbers with a total of 100, i.e. 32 + ? = 100.	**Expanded horizontal addition:** • Teachers model how numbers can be partitioned into tens and ones, including different ways, e.g. 36 = 30 + 6 36 = 20 + 10 + 6 • Add numbers using structured apparatus to support understanding of place value. • Make connections between partitioning both numbers using structured apparatus and partition the second number only using a number line.	 Add…  …and…  By partitioning and recombining 30+ 40 = 70 5 + 7 = 12 70 + 12 = 82 

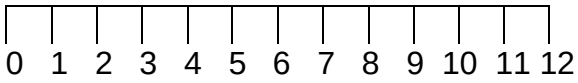
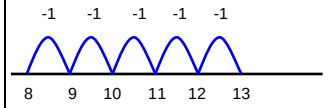
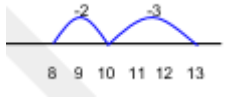
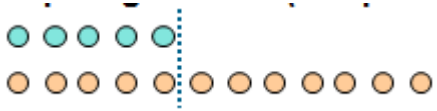
Stage 4:	Continue practising previous skills. Count forwards and backwards from 0 in multiples of 6, 7, 9, 25 and 1000 using counting sticks, number lines, number squares, etc. Count up and down in tenths, hundredths and simple fractions using models and images, plus Dienes / pixie Dienes equipment and a counting stick.	Bridging through 60 for time, i.e. 70 minutes = 1 hour and 10 minutes. Rounding any number to the nearest 10, 100 or 1000. Rounding numbers with one decimal place to nearest whole number. Explore inverse as a way to derive new facts and to check accuracy of answers.	As above. Use known facts and place value to derive new ones, i.e. 'If I know 8 + 3 = 11, I also know 0.8 + 0.3 = 1.1 and 8/100 + 3/100 = 11/100.' Sums and differences of pairs of multiples of 10, 100 or 1000. Addition doubles of numbers to 100. Pairs of fractions totalling one.	**Expanded horizontal method, leading to columnar addition:** • Written recording should follow teacher modelling around the size of numbers and place value using a variety of concrete/pictorial materials, e.g. Numicon shapes, Dienes and place-value cards. • As children move towards using a columnar method, links continue to be made with earlier models and images, including the number line.	 a  b  c  d

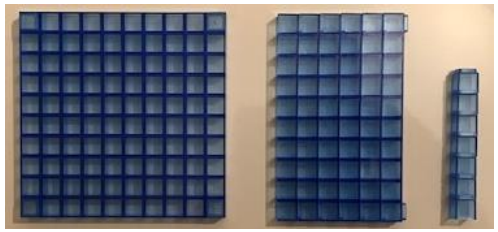
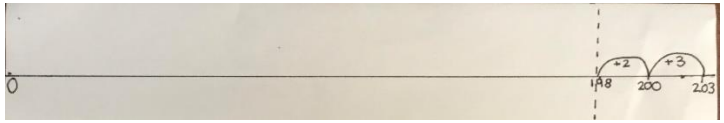
					It is important that empty number lines are kept as well as using more formal written calculation methods. Counting on more efficiently: 
Stage 5:	Count forwards and backwards in steps of powers of 10 for any given number up to one million. Continue to count forwards and backwards in simple fractions. Count forward and backwards in appropriate decimals and percentages.	Use apparatus and knowledge of place value to add decimals, i.e. 3.4 + 2.5 = 5 + 0.9  Reorder increasingly complex calculations, i.e. 1.7 + 2.8 + 0.3 = 1.7 + 0.3 + 2.8 Compensating – i.e. 405 + 399 → add 400 and then subtract one.	Continue to practice previous stage and make links between known facts and addition pairs for fractions, percentages and decimals Doubles and halves of decimals, i.e. half of 5.6, double 3.4. Sums and differences of decimals, i.e. 6.5 + 2.7	**Expanded vertical method, leading to columnar addition:** • Teachers model a column method that records and explains partial mental methods. • There remains an emphasis on the language of calculation, e.g. 'Forty plus seventy equals one-hundred and ten.'… 'Seven add six equals thirteen.' …before recombining numbers. Teachers also model the language of: 'Four tens add seven tens total eleven tens or 110.' • Teachers similarly advance to model the addition of two 3-digit numbers with the expectation that as children's knowledge of place value is secured, they become ready to approach a formal compact method.	Informal columnar: Adding the hundreds first: 471 + 356 700 120 7 827 Adding the ones first: 471 + 356 7 120 700 827 

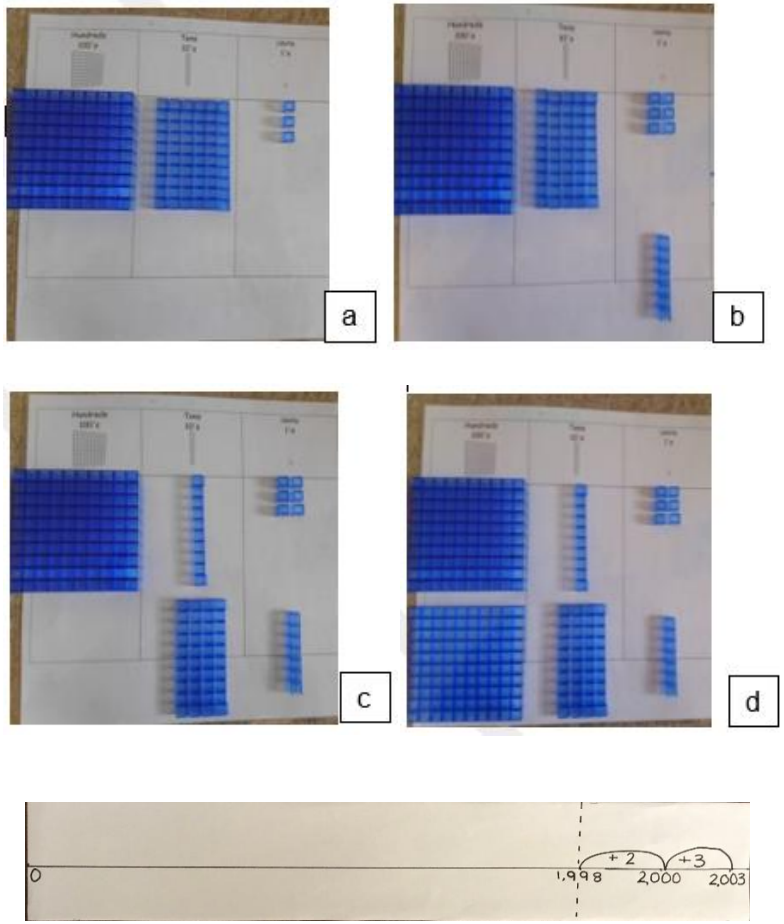
Stage 6:	Continue to practice previous skills. Count forwards and backwards in simple fractions, decimals and percentages.	Bridging through decimals, i.e. 0.8 + 0.35 = 0.8 + 0.2 + 0.15 using empty number lines. Partitioning using near doubles, i.e. 2.5 + 2.6 = 5 + 0.1 Reorder decimals, i.e. 4.7 + 5.6 – 0.7 …as… 4.7 – 0.7 + 5.6 = 4 + 5.6.	Using children's confident recalling of basic facts to 20/100 and deriving facts using place value, make links between decimals, fractions and percentages. i.e. 1 + 19 10 + 190 100 + 1900 Question: What do you notice?	**Columnar addition (formal written method):** • The concept of exchange is reinforced through continued use of manipulatives. • Teachers model: 1. "I have two tens and five ones, which need adding to four tens and seven ones." 2. "I add five ones to seven ones, which gives me twelve ones." 3. "I exchange ten of my twelve ones for a ten counter." 4. "I add my three tens and four tens to make seven tens." "Altogether, I have seven tens and two ones." • Teachers similarly advance to model the addition of two 3-digit numbers and then go beyond. 587 + 475 1062 1 1	**Pupils to be encouraged to consider mental strategies first.** Formal columnar – using an example with smaller value numbers to exemplify: 

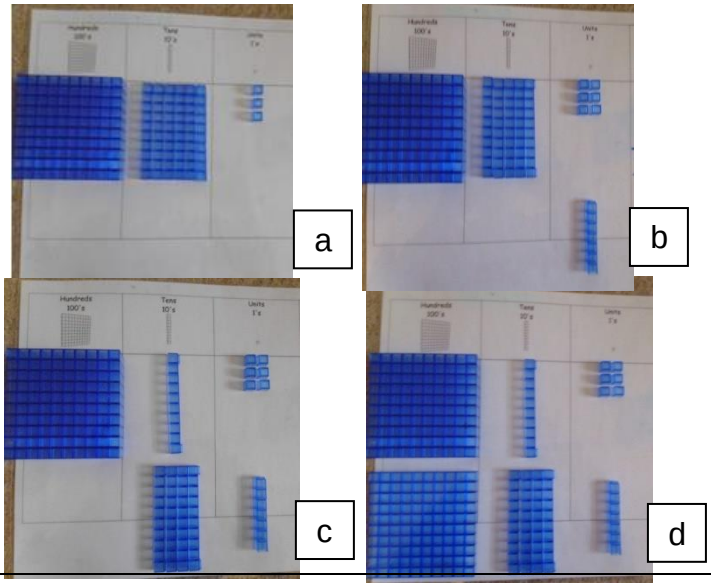
Subtraction:

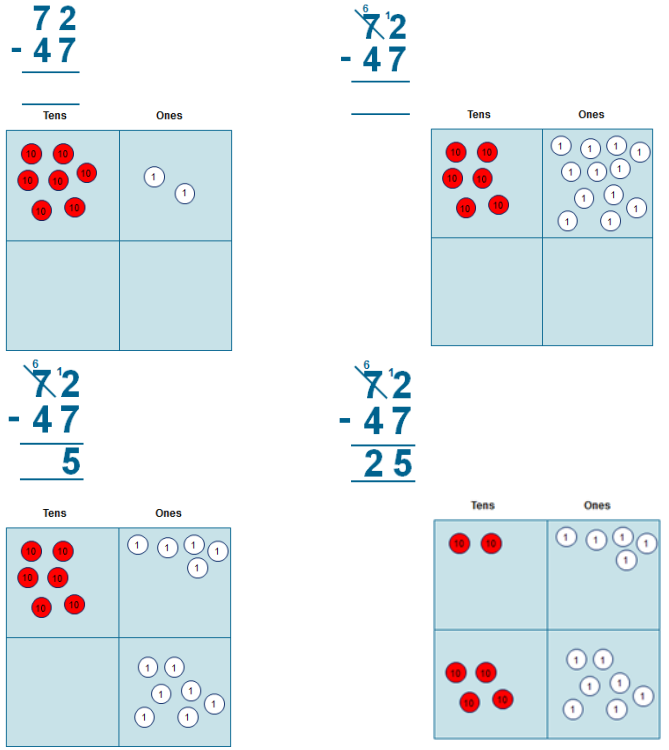
	Counting	Mental strategies	Rapid Recall	Written calculation and appropriate models and images to support conceptual understanding	
Stage 1:	Count in ones to and across 100, forwards and backwards starting from 0, 1 and other numbers. Count in multiples of two, five and ten.	<u>Explicitly teach every mental maths strategy detailed above.</u> Pupils use apparatus to explore addition as the inverse of subtraction:  'One less than five is four' 	Rapid recall of subtraction facts for numbers up to 10. Use structured apparatus, i.e. Numicon, tens frames, abaci etc. 	**Subtraction as taking away from a group:** • Teachers model how to remove counters/objects and count back on a number track. This is a precursor to use of a fully numbered number-line. **Whole / part-whole model:** • The concept of a whole / part-whole model is introduced.	 'Five minus two totals three'  'Six take away two leaves four'  'One less than six is five'  Tens frame  Bar model  Cherry model

Stage 2:	Continue practising above skills. Count in steps of 2, 3 and 5, forwards and backwards to and from zero. Count in tens from any number – link to coins in a piggy bank as well as a number square.	<u>Explicitly teach every mental maths strategy detailed above.</u>	Recall subtraction (and addition) facts for all numbers to 20.	**Taking away:** • Children begin to use number lines to support their own calculations, initially counting back in ones before beginning to work more efficiently. **Finding the difference:** • Teachers model how to find the difference when two numbers are relatively 'close together.'	Number line with all numbers labelled  13 – 5 = 8  to 13 – 5 = 8  Comparing two sets to find the difference. 

Stage 3:	Continue practising above skills. Count from 0 in multiples of 4, 8, 50 and 100. Count on and back by 10 or 100 from any two-digit number. Link to counting stick counting forwards and backwards flexibly. Count up and down in tenths – linking to visual image.	Reinforce partitioning and bridging through multiples of 10, plus adjusting when subtracting 11 or 9. Use structured apparatus to understand that subtraction undoes addition and link with inverse number operations.	Connect subtractions from ten to subtractions from multiples of 10 totalling 100.  Use 10ps in tens frame. Subtract two digit numbers from 100 i.e. ? = 100 - 78	**Taking away:** • When teaching children about reduction, highlight the importance of only partitioning one number. **Finding the difference:** • Children move on to find the difference by making number line comparisons.	Subtraction by partitioning with use of manipulatives and linked with a horizontal expanded written algorithm: 167 – 24 = 143 20 4  In either order… To begin: 167 – 20 = 147  Then: 147 – 4 = 143  100 + 60 + 7 - 20 + 4 0 + 40 + 3 Finding the difference on a number line:  Children should note that finding the difference is often the most efficient way of solving a subtraction problem when two numbers are close together. e.g. 61 – 59

Stage 4:	Continue practising of previous skills. Count forwards and backwards from 0 in multiples of 6, 7, 9, 25 and 1000 using counting sticks, number lines, number squares, etc. Count up and down in tenths, hundredths and simple fractions using models and images, i.e. Dienes / Pixie Dienes equipment, counting stick, ITPs.	Bridging through 60 for time, i.e. 70 minutes = 1 hour and 10 minutes Rounding any number to the nearest 10, 100 or 1000. Rounding numbers with one decimal place to nearest whole number. Explore inverse as a way to derive new facts and to check accuracy of answers.	As above. Use known facts and place value to derive new ones, i.e. 'If I know 11 - 3 = 8, I also know 1.1 - 0.3 = 0.8 and 8/100 - 3/100 = 5/100.' Sums and differences of pairs of multiples of 10, 100 or 1000. Subtraction of fractions totalling 1, i.e. 1 – 0.3 = 0.7	**Taking away:** • Subtraction by partitioning with use of manipulatives, and including transfer / exchange, linked with a horizontal expanded written algorithm in preparation for a future formal column method. **Finding the difference:** • Finding the difference continues to be highlighted where the two numbers are close together – using a number line on a strip of paper.	363 – 147 = 216 50 13 300 + ~~60~~ + ~~3~~ - 100 + 40 + 7 200 + 10 + 6 

Stage 5:	Count forwards and backwards in steps of powers of 10 for any given number up to one million. Continue to count forwards and backwards in simple fractions. Count forward and backwards in appropriate decimals and percentages.	Use apparatus and knowledge of place value to subtract decimals, i.e. 3.8 - 2.5 = 1.3 Reorder increasingly complex calculations, i.e. 1.7 – 0.5 – 0.7 = 1.7 – 0.7 – 0.5. Compensating – i.e. 405 - 399 → subtract 400 and then add 1.	Continue to practise previous stage and make links between known facts and addition pairs for fractions, percentages and decimals. Doubles and halves of decimals, i.e. half of 5.6, double 3.4. Sums and differences of decimals, i.e. 6.5 + 2.7	**Column method with Dienes:** • Subtraction by partitioning with use of manipulatives, and including transfer / exchange, linked with a formal column written algorithm.	5 1 3 ~~6~~ 3 - 147 216  a b c d

Stage 6:	Continue to practise previous skills. Count forwards and backwards in simple fractions, decimals and percentages.	Bridging through decimals, i.e. 1.5 – 0.8 = 1.5 – 0.5 then – 0.3 using empty number line.	Using children's confident recalling of basic facts to 20/100 and using place value, make links between decimals, fractions and percentages. 19 – 1 = 190 – 10 = 1900 – 100 = 1.9 – 0.1 = Question: What do you notice?	**Column method with place value counters:** • The concept of transfer / exchange is continued through use of manipulatives. • Teachers model: 1. "I have seven tens and two ones. I need to subtract four tens and seven ones." 2. "At the moment, I cannot subtract seven ones from two ones, so I need to transfer one ten to become ten ones." 3. "Now I can take away seven ones from twelve ones, so that I have fives ones left." 4. "I can now subtract four tens from six tens, which leaves me with two tens." 5. "I recombine two tens and fives ones to understand that I am left with twenty-five."	**Pupils to be encouraged to consider mental strategies first.** Formal columnar – using an example with smaller value numbers to exemplify: 

Even when a child reaches stage six, they are still taught that, for some calculations and in some contexts, one of the earlier methods remains the most efficient. For example, 2007-1992 is easier to solve by counting on using a number line than to try to solve using the column method. We encourage children to make informed decisions about which strategies to use, based on their secure understanding of these different methods.

End-of-year expectations:
The National Curriculum specifies what most children can be expected to achieve at the end of each year group. However, children should progress through the stages when they are ready and they may not all progress at the same rate.

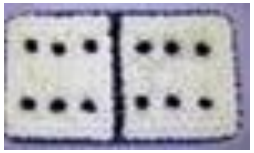
It is expected that the vast majority of pupils progress at the following pace:

- Year 1: Add and subtract one– and two-digit numbers to 20
- Year 2: Add and subtract TU + U, TU + multiples of ten, TU + TU, U + U + U
- Year 3: Add and subtract 3-digit numbers to ones, tens and hundreds
- Year 4: Add and subtract up to 4-digit numbers using column methods where appropriate
- Year 5 and Year 6: Add and subtract numbers with up to 4 digits, including using formal written methods

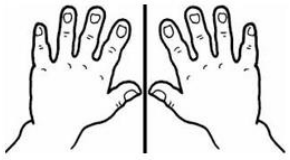
Mental calculation strategies for multiplication and division:

Doubling and halving:

Double six is 12…



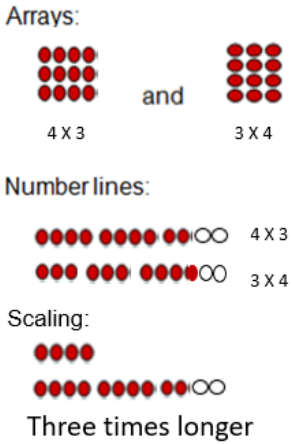
Double five is ten…



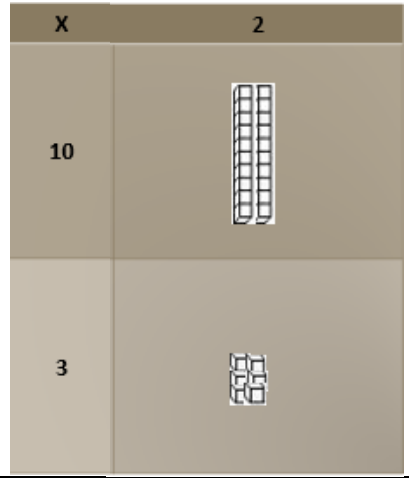
Double 16 can be calculated by working out…
Double ten → 20
Double six → 12

With links to finding four-times a given value and finding a quarter of a value.

Knowing multiplication and division facts to 12 X 12:



Multiplying a teen number by one-digit number:



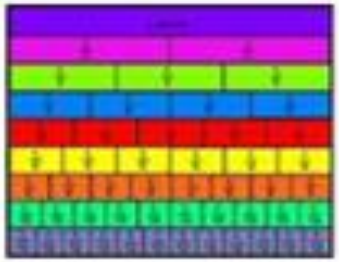
Multiplying and dividing by multiples of ten:

20 X 10 = 200

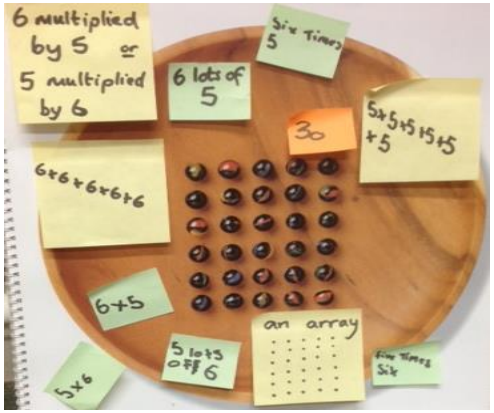
Hundreds	Tens	Ones
	2	0

'Add a place value holder'

Identifying fractions, decimals and percentages:

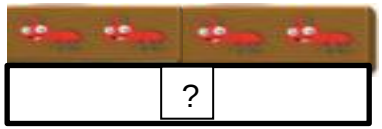
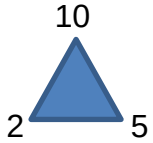
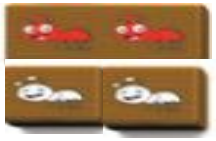
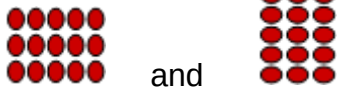
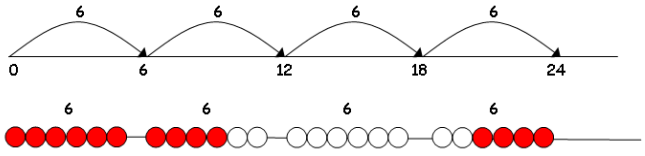


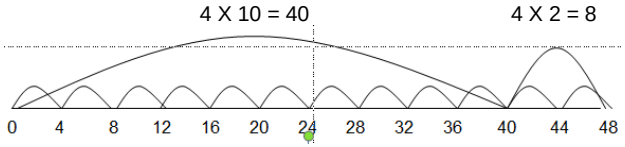
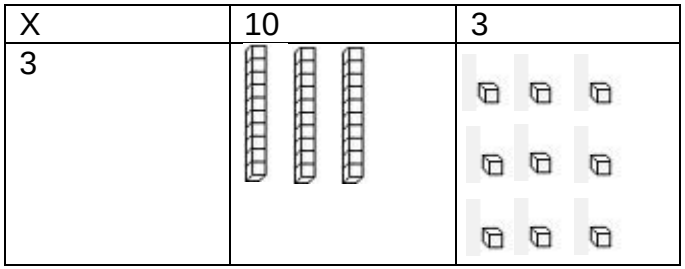
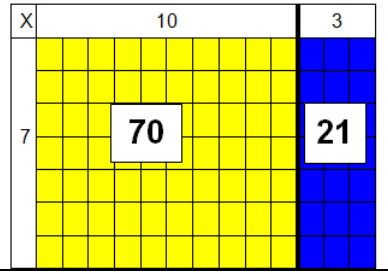
Milk the maths…



...by allowing children to make connections between number facts.

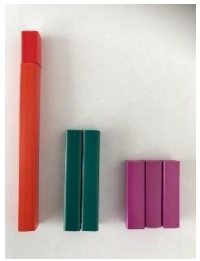
Multiplication:

	Counting	Mental strategies	Rapid recall	Written calculation and appropriate models and images to support conceptual understanding	
Stage 1:	Count forwards and backwards in 2s, 5s and 10s	Doubling up to six and then ten whilst using related models and images.	Derive/recall doubles up to five and derive/recall halves up to ten.	**Developing early conceptual understanding of multiplication (grouping):**	Use objects, pictorial representations and arrays to show the concept of multiplication:  Early bar model 
Stage 2:	Count forwards and backwards in 2s, 3s, 5s and 10s from zero.	Begin to understand and use inverse number operations:  Stories are used alongside a triad to help children understand links between number operations, e.g. “There are five pencils in two packs, which means that there are ten pencils altogether.” Doubling is reinforced using a whole/part-whole model: 	Derive/recall doubles up to ten and derive/recall halves up to twenty. Recall & use multiplication facts for the 2X, 5X and 10X-tables. Learn what happens when a number is multiplied by zero or one.	**Understanding multiplication as repeated addition:** • Investigate multiplication as repeated addition, so that the law of cummutativity is understood. • Whilst arrays are also modelled explicitly at this stage, it is important to note that they will continue to be a key model at later stages, alongside more formal methods of calculation.	Arrays: 5 X 3 and 3 X 5  Connect related facts with both array and repeated addition images. Repeated addition on the number line linked with manipulatives: 6 X 4 = 24  So: ‘Six multiplied by four’ …or… ‘Six taken four times.’

Stage 3:	Counting forwards and backwards in 2s, 3s, 4s, 5s, 8s and 10s from zero. Count up and down in tenths.	Use doubling to make connections between the 2X, 4X and 8X-tables. Understand that multiplication can be undertaken by partitioning numbers, e.g. 12 X 4 = 10 X 4 + 2 X 4 Introduce the structure of scaling: e.g. Find a ribbon that is 4 times as long as the blue ribbon  2cm 8cm	Recall and use multiplication facts for the 2X, 3X, 4X, 5X, 8X and 10X tables.	**Relate multiplying a 2-digit by 1-digit number using repeated addition and arrays to represent:**	Children use an empty number line to group efficiently: 4 X 12 = 48  3 X 13 = 39  7 X 13 = 91 

<table>
<tr>
<td>Stage 4:</td>
<td>Counting forwards and backwards in 2s, 3s, 4s, 5s, 7s, 8s, 10s, 25s and 1000s from zero.

Count up and down in tenths and hundredths.</td>
<td>Derive factor pairs of numbers using models and images, e.g. Cuisenaire
1 and 12
2 and 6
3 and 4

Use reordering to multiply three numbers.

Children learn about the associative law:
(9 X 5) X 10 = (10 X 5) X 9</td>
<td>Recall & use multiplication facts for all times-tables up to 12 X 12.</td>
<td>Relate multiplying a 3 or 2-digit by 1-digit number with arrays towards using long/short multiplication:</td>
<td>Relate multiplying a 3/2-digit by 1-digit number, whilst refining the written notation used.

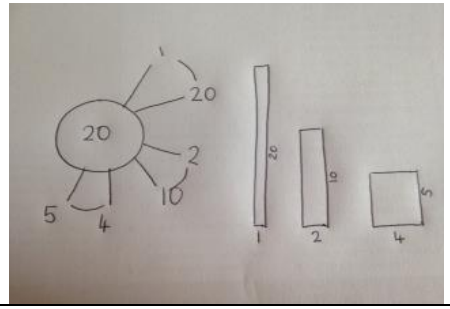
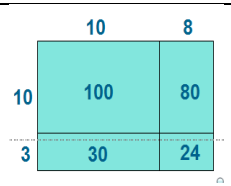
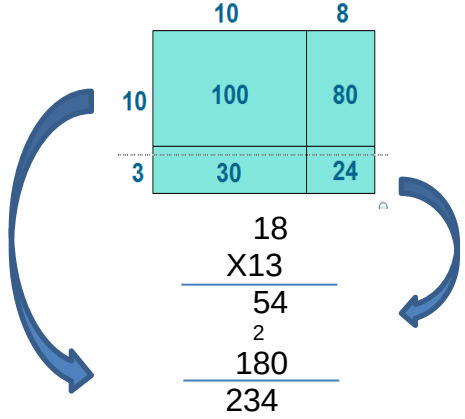
114 X 2 = 228

<table>
<tr><td>X</td><td>100</td><td>10</td><td>4</td></tr>
<tr><td>2</td><td>  </td><td>  </td><td>  </td></tr>
</table>

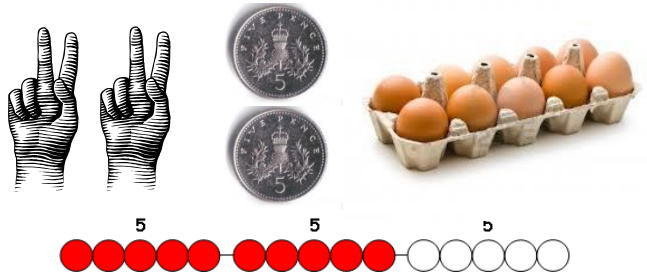
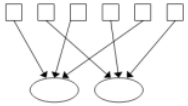
114 X 2 =
100 X 2 = 200
10 X 2 = 20
4 X 2 = 8
= 228

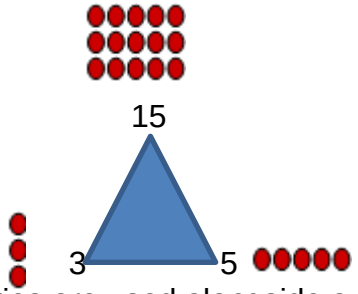
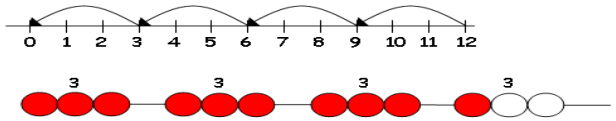
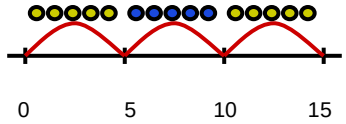
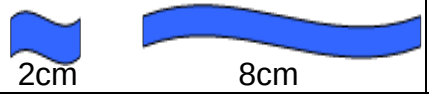
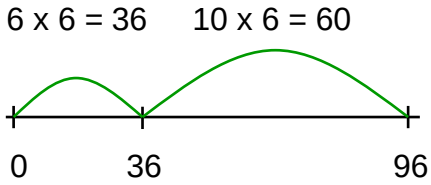
Link with distributive law:
(100 X 2) + (10 X 2) + (4 X 2) = 228

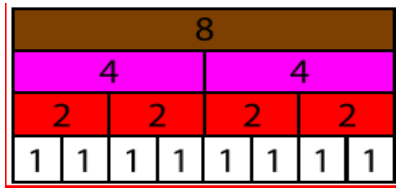
At this stage, the non-statutory guidance in the National Curriculum suggests teaching short multiplication; however, the team feel that an expanded form of calculation (as set out above) is be a better lead into long/short multiplication.</td>
</tr>
</table>

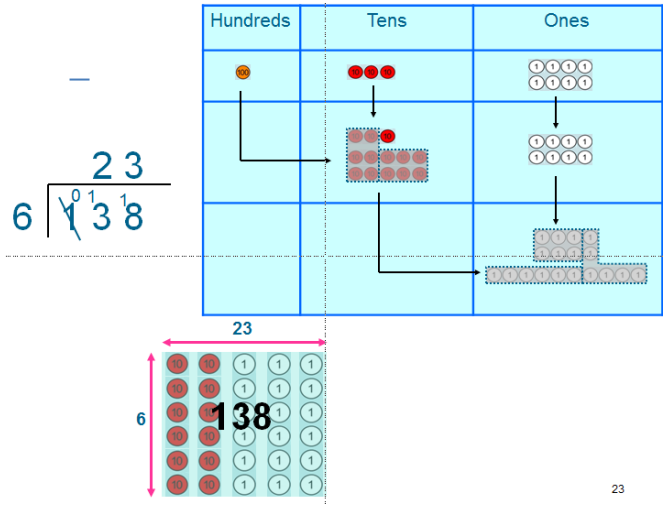
Stage 5:	Counting forwards and backwards in 2s, 3s, 4s, 5s, 6s, 7s, 8s, 9s, 10s, 25s and 1000s from zero.	Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers. 	Recall & use multiplication facts for all times-tables up to 12 X 12.	**Relate multiplying a 4/3/2-digit by 1/2-digit number with grid to using long multiplication:**	 18 X13 24 30 80 100 234
Stage 6:	Consolidate all previous counting, including forwards and backwards in fractions.	Perform mental calculations, including with mixed numbers and operations.	Recall & use multiplication facts for all times-tables up to 12 X 12.	**Relate multiplying a 4/3/2-digit by 1/2-digit number with grid to using short multiplication:**	 18 X13 54 2 180 234 Once children have fully grasped the concept of multiplication alongside manipulatives and an expanded written method, they will be well-placed to progress towards a more compact written algorithm.

Division:

	Counting	Mental strategies	Rapid recall	Written calculation and appropriate models and images to support conceptual understanding	
Stage 1:	Count forwards and backwards in 2s, 5s and 10s	Doubling up to six and then ten whilst using related models and images.	Derive/recall doubles up to five and derive/recall halves up to ten.	**Developing early conceptual understanding of division as grouping and sharing:**	Use objects, pictorial representations and arrays to show the concept of division as grouping and sharing.  "Two children share six pencils between them"  "Six children are asked to get into three equal groups" 

Stage 2:	Count forwards and backwards in 2s, 3s, 5s and 10s from zero.	Begin to understand and use inverse number operations.  Stories are used alongside a triad to help children understand links between number operations, e.g. “15 children are asked to get into three groups and find out that there are five people in each group.”	Derive/recall doubles up to ten and derive/recall halves up to twenty. Recall and use multiplication facts for the 2X, 5X and 10X-tables.	**Understanding division as repeated subtraction:** • Investigate division as repeated subtraction. • Through teacher modelling, children need to know that division is not commutative.	Number lines: 12 ÷ 3 = 4  15 ÷ 5 = 3  Early bar model 
Stage 3:	Counting forwards and backwards in 2s, 3s, 4s, 5s, 8s and 10s from zero.	Use doubling to make connections between the 2X, 4X and 8X-tables. Understand that multiplication can be undertaken by partitioning numbers, e.g. 12 X 4 = 10 X 4 + 2 X 4 Introduce the structure of scaling: e.g. Find a ribbon that is 4 times as long as the blue ribbon.  	Recall & use multiplication facts for the 2X, 3X, 4X, 5X, 8X and 10X tables.	**Dividing a 2-digit by 1-digit number, representing this efficiently on a number line:**	Children use an empty number line to chunk efficiently. 96 ÷ 6 = 16  Conceptual understanding can be provided through use of a bead string to highlight the chunks. 

Stage 4:	Counting forwards and backwards in 2s, 3s, 4s, 5s, 7s, 8s, 10s, 25s and 1000s from zero.	Derive factor pairs of numbers using models and images, e.g. Cuisenaire.  	Recall & use multiplication facts for all times-tables up to 12 X 12.	**Dividing a 3 or 2-digit by 1-digit number, representing this efficiently on a number line, also in relation to long division:** • At this stage, remainders may be present in a practical context.	Children use an empty number line to chunk efficiently. 224 ÷ 8 = 28 8 x 8 = 64 20 x 8 = 160  28 8 224 - 160 (8 X 20) 64 - 64 (8 X 8) 0 …or… 28 8 224 20 X 8 = 160 64 8 X 8 = 64 0
Stage 5:	Counting forwards and backwards in 2s, 3s, 4s, 5s, 6s, 7s, 8s, 9s, 10s, 25s and 1000s from zero.	Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.	Recall & use multiplication facts for all times-tables up to 12 X 12.	**Dividing a 4/3/2-digit by 1-digit number, in relation to long division:** • By this stage, there is a statutory requirement that children can use a formal written calculation method, such as long division. • Short division may begin to be taught alongside long division, but still with use of visual representations	As schools have autonomy to decide children's progression in learning between long and short division in Years 5 and 6, the maths team suggest beginning with long division. Remainders should be interpreted in the following ways when long division is used: • as whole numbers • as fractions • through rounding in an appropriate way to the context Long division: 415 ÷ 9 = 46 and 1/9 46 and 1/9 9 415 40 X 9 = 360 55 6 X 9 = 54 1

Stage 6:	Consolidate all previous counting, including forwards and backwards in fractions.	Perform mental calculations, including with mixed numbers and different number operations.	Recall & use multiplication facts for all times-tables up to 12 X 12.	**Dividing a 4/3/2-digit by 2/1-digit number, in relation to long and then short division:** • By this stage, there is a statutory requirement that children can use formal written calculation methods, including long and short division. • Use of visual representations – like the ones opposite – remain important.	As schools have autonomy to decide children's progression in learning between long and short division in Years 5 and 6, the maths team suggest moving from long division to short division. Remainders should be interpreted in the following way when short division is used: • through rounding in an appropriate way to the context Long division: 432 ÷ 15 = 28 4/5 28 15 ⟌432 20 X 15 = 300 132 8 X 15 = 120 12 $\frac{12}{15} = \frac{4}{5}$ Answer: 28 $\frac{4}{5}$ Short division: 138 ÷ 6 = 23 

					Key language: 'How many groups of six one-hundreds are there in one-hundred?' 'How many groups of six tens are there in thirteen tens?' 'How many groups of six ones are there in eighteen?'

End-of-year expectations:

The National Curriculum specifies what most children can be expected to achieve at the end of each year group. However, children should progress through the stages when they are ready and they may not all progress at the same rate.

It is expected that the vast majority of pupils progress at the following pace:

- Y1: Solve one step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
- Y2: Solve problems involving multiplication using materials, arrays, mental methods and multiplication facts.
- Y3: Multiply 2 digits by 1 digit, using mental and progressing to formal written methods.
- Y4: Multiply 2 digits by 1 digit using formal written layout. Multiply 3 digits by 1 digit using formal written layout.
- Y5: Multiply numbers up to 4 digits by a 1 digit number using the formal written method of short multiplication. Multiply numbers up to 4 digits by a 2 digit number using the formal written method of long multiplication. Multiple whole numbers and those involving decimals by 10, 100, 1000
- Y6: Multiply up to 4 digits by 2 digits using the formal written method of long multiplication. Multiply numbers by 10,100, 1000 giving answers up to 3 decimal places.